Nontopological structures in the baby-Skyrme model

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Abstract

We report our observations that the baby-Skyrme model in (2+1) dimensions possesses non-topological stationary solutions which we call psedo-breathers. We discuss their properties and present our results on their interaction with the topological skyrmions.

1 Introduction.

The $(2+1)$-dimensional baby-Skyrme field theory model is described by the Lagrangian density

$$L = F_\pi\big(\frac{1}{2}\partial_\alpha\vec{\phi}\partial^\alpha\vec{\phi} - \frac{k^2}{4}(\partial_\alpha\vec{\phi}\times\partial_\beta\vec{\phi})(\partial^\alpha\vec{\phi}\times\partial^\beta\vec{\phi}) - \mu^2(1-\vec{n}\cdot\vec{\phi})\big). \qquad (1)$$

Here $\vec{\phi} \equiv (\phi_1, \phi_2, \phi_3)$ denotes a triplet of scalar real fields which satisfy the constraint $\vec{\phi}^2 = 1$; $(\partial_\alpha\partial^\alpha = \partial_t\partial^t - \partial_i\partial^i)$. As mentioned in [1]-[2] the first term in (1) is the familiar Lagrangian density of the pure S^2 σ model. The second term, fourth order in derivatives, is the (2+1) dimensional analogue of the Skyrme-term of the three-dimensional Skyrme-model [3]. The last term is often referred to as a potential term. The last two terms in the Lagrangian (1) are added to guarantee the stability of the skyrmion[4].

This model has static solutions which describe topologically stable field configurations called skyrmions[5]. Such solutions have been studied in detail; results can be found in [5] and [1]. However, this model also has many other

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interesting solutions. In particular, it has plane wave like solutions. Moreover, any solution of the Sine-Gordon model is also a solution of the baby-Skyrme model. To see this observe that if we parametrise $\vec{\phi}$ as

$$\vec{\phi} = (\sin f \cos\psi, \sin f \sin\psi, \cos f), \tag{2}$$

and then assume that ψ is constant the equation of motion for the field f reduces to

$$\partial_\mu \partial^\mu f + \mu^2 \sin(f) = 0, \tag{3}$$

which, of course, is the sine-Gordon equation. Thus all solutions of this equation, kinks, breathers *etc* are automatically solutions of the baby-Skyrme model.

However, as we have shown in [6] all such solutions are unstable. Any small perturbation destroys them. For any of these solutions we have found that as soon as some region of the wave is perturbed, the wave collapses around this point, emitting radiation. The collapse front then propagates rapidly along the solitonic wave destroying it completely.

While performing simulations with large amplitude breather waves we have observed the formation of a radially symmetric breather-like soliton. As this solution seemed very similar to the pulsons observed by [7] we have decided to look at it in more detail.

2 Pseudo-breathers

Looking at the time evolution of the field configurations seen in our simulations has lead us to make the following ansatz for our pseudo-breather fields:

$$\vec{\phi} = (\sin f(r,t), 0, \cos f(r,t)). \tag{4}$$

Then, simple calculations show that the equation of motion reduces to the radial sine-Gordon equation:

$$f_{tt} - f_{rr} - \frac{f_r}{r} + \mu^2 \sin(f) = 0. \tag{5}$$

This equation has already been studied in [7], where it was shown that it had time dependent solutions similar to a breather, but which radiate their energy and slowly die out. The authors of [7] called such configurations pulsons.

In [8] we have also shown that there exist stable time dependent solutions of (5). Radial field configurations of (5) radiate relatively quickly when their amplitude of oscillation is relatively small, *ie* when the value of f never becomes larger than $\pi/2$ at the origin (these are the pulsons studied in [7].) However, when the amplitude of oscillation is larger than $\pi/2$ the general field configuration radiates its energy very slowly asymptotically reducing the amplitude of oscillation to a value a little larger than $\pi/2$ and settles at a pseudo-breather with a period of oscillation $T \sim 20.5$ (when $\mu^2 = 0.1$). By trial and error we have found that

$$f(r,0) = 4\,atan(C\,exp(-\frac{2}{\pi}\frac{\mu r}{K}atan(\frac{\mu r}{K}))) \tag{6}$$

and $\frac{\partial f}{\partial t}(r,0) = 0$ with $K = 10$ and $C = \tan(\pi/8)$ is a good initial condition which leads to this metastable pseudo-breather solution.

We have checked that this pseudo-breather solution is stable when enbedded into the S^2 model.

Its energy is given by

$$E_{PB} \sim 3.97$$

which shows that it is approximately 2.5 heavier than the baby-skyrmion. Moreover, its topological charge density is identically zero thus it has enough energy to decay into a skyrmion anti-skyrmion pair. However, we have not seen such a decay in our simulations.

As our pseudo-breather can only be determined numerically, in practice we have to use the field configurations which we have found in our numerical simulations. The excess of energy over the final configuration can then be seen as an excitation energy which is slowly radiated away. The scattering properties of pseudo-breathers are quite interesting. When the pseudo-breathers are embedded into the baby-skyrmion model the field configurations have an extra degree of freedom corresponding to their orientation in the ϕ_1, ϕ_2 plane. When two pseudo-breathers are set at rest near each other, the force between them depends on their relative orientation: when they are parallel to each other and oscillate in phase, they attract each other, overlap and form a new structure which appears to be an exited pseudo-breather. This pseudo-breather then slowly radiates away its energy. The non-topological nature of pseudo-breathers allows them to merge and form a new structure of the same type.

If the two pseudo-breathers are anti parallel, *ie* if they oscillate completely out of phase, then the force between them is repulsive. When the two pseudo-breathers have a different orientation they slowly rotate themselves until they become parallel; then they move towards each other and form an exited pseudo-breather structure.

When two pseudo-breathers are sent towards each other with some kinetic energy, the scattering is more complicated. Depending on the initial speed or the scattering impact parameter, they either merge into a single pseudo-breather or they undergo a forward scattering. The details of these scattering properties are given in [8].

When a skyrmion and a pseudo-breather are put at rest next to each other the overall interaction between them makes the skyrmion slowly move away from the pseudo-breather while the pseudo-breather looses some of its energy faster than when it is placed there by itself.

To scatter a skyrmion with a pseudo-breather we have placed the pseudo-breather soliton at rest, and we have sent the skyrmion towards it. We have performed this scattering for different orientations of the pseudo-breather, for different values of the impact parameter and for different speeds.

The results of our simulations are reported in [6]. We have found that the amount of energy lost by the pseudo-breather during the scattering is larger when the overlap between the skyrmion and the pseudo-breather, both in time and space, is greater. In some cases the pseudo-breather is completely destroyed by the collision. The oscillation of the pseudo-breather makes the interaction time dependent and, as a result, the pattern of scattering angles observed in the simulation is quite complicated.

3 Conclusions

We have shown that the baby-Skyrme model, in addition to skyrmions has other solutions which are nontopological in nature and periodic in time. These new solutions have interesting scattering properties with each other and with skyrmions. They are stable but can be destroyed if perturbed too much.

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References

[1] B.M.A.G. Piette, B.J. Schroers and W.J. Zakrzewski, Nucl. Phys. B **439** (1995) 205

[2] B.M.A.G. Piette, H.J.W. Muller-Kirsten, D.H. Tchrakian and W.J. Zakrzewski, Phys. Lett. B **320** (1994) 294

[3] T.H.R. Skyrme, Proc. Roy. Soc. A**260** (1961) 125; G.S. Adkins, C.R. Nappi and E. Witten, Nucl. Phys. B**228** (1983) 552

[4] R.A. Leese, M. Peyrard and W.J. Zakrzewski, Nonlinearity **3** (1990) 773

[5] B.M.A.G. Piette, B.J. Schroers and W.J. Zakrzewski, Z. Phys. C **65** (1995) 165

[6] A. Kudryavtsev, B. Piette, W.J. Zakrzewski, Durham Univ. preprint DTP-96/17.

[7] P.L. Christiansen and P. Lomdahl, Physica **2**D (1981) 482

[8] B. Piette and W.J. Zakrzewski, Durham Univ. preprint DTP-97/15